STAT 8810 Assignment 1

Due: September 13th, 2017

	Team Member	Team Member	Team Member
team 1	1	8	
team 2	7	6	
team 3	9	4	
team 4	2	5	3

Question 1

Write a function to generate an unconditional realization of a Gaussian Process with Gaussian correlation function in p-dimensions where the observations are located on a equally space grid of $N = n^p$ locations. Generate a realization in dimension $p = 2$ at $N = 30^2$ locations.

Question 2

(a) Plot the theoretical variograms for the Gaussian, exponential and spherical models and summarize any important differences.

(b) Generate a GP realization using the Gaussian model at $N = 100$ locations in 2-D and fit the Gaussian and Exponential variogram models to the data. Perform the same exercise for a GP realization generated using the Exponential model.

Question 3

Suppose the process $Z(x), x \in \mathbb{R}^2$ is a GP with Gaussian correlation function, mean zero and constant variance σ^2. Derive the covariance formulas for $Cov(Z(x), Z_1(x'))$ and $Cov(Z_1(x), Z_2(x'))$, where $Z_1(x) = \frac{\partial}{\partial x_1} Z(x)$ and $Z_2(x) = \frac{\partial}{\partial x_2} Z(x)$. Write a function to generate a 1-D unconditional realization of a Gaussian Process and its first derivative using the Gaussian correlation model. Verify the derivative numerically by performing a finite-differences approximation of the derivative from a realization of the process using $Z'(a) \approx \frac{Z(a+h)-Z(a)}{h}$.

Question 4

Consider the correlation function

$$\rho(h) = \begin{cases} 1 - \frac{||h||}{\phi}, ||h|| \leq \phi \\ 0, \text{ otherwise} \end{cases}$$

This is a valid correlation function in 1-D. Show it is not valid in 2-D. Hint: Consider points s_{ij} at a 6×8 grid of size $\frac{\phi}{\sqrt{2}}$. Look at $\text{Var}\left(\sum_{i,j} a_{ij} Z(s_{ij})\right)$ where $a_{ij} = 1$ if $i + j$ is even and $a_{ij} = -1$ otherwise.

Question 5

Consider a stationary random field $z(s)$ for $s \in D \subseteq \mathbb{R}^2$ observed at sites $s_1, \ldots, s_n$.

(a) Derive the unbiased linear predictor with smallest variance.

(b) Prove that your predictor interpolates the observed data.

(c) Prove that the squared prediction error of your interpolator is zero at the observed data locations.