STAT 8810 Assignment 2
Due: October 9th, 2017

	Team Member	Team Member
team 1	9	3
team 2	1	8
team 3	7	4
team 4	2	6

Question 1

(a) Write a function that calculates the Maximum Likelihood Estimates for the parameters of a Gaussian Process with constant mean μ, variance σ^2 and using the Gaussian correlation function.

(b) Using your function, fit the GP model to Frankes Function at the $N = 15$ settings of the inputs in frankesinputs.dat (see `https://www.sfu.ca/~ssurjano/franke2d.html` for the function definition).

(c) Plot the BLUP over a fine grid of points along with a 95% prediction interval.

(d) Plot the likelihood surface as a function of the two correlation parameters over a dense grid and highlight the MLE estimates.

Question 2

Perform a simulation study that investigates the empirical coverage of the GP prediction intervals when

(a) a 1D GP model with Gaussian correlation and $\rho = 0.9$ is fit to data generated from a 1D GP model with Gaussian correlation and $\rho = 0.005$, and

(b) a 1D GP model with Gaussian correlation and $\rho = 0.005$ is fit to data generated from a 1D GP model with Gaussian correlation and $\rho = 0.9$.

Question 3

Perform a simulation study that investigates the empirical coverage of the GP prediction intervals when

(a) a 1D GP model with Exponential correlation and $\rho = 0.2$ is fit to data generated from a 1D GP model with Gaussian correlation and $\rho = 0.2$, and

(b) a 1D GP model with Gaussian correlation and $\rho = 0.2$ is fit to data generated from a 1D GP model with Exponential correlation and $\rho = 0.2$.

Question 4

Perform a simulation study that investigates the empirical coverage of the GP prediction intervals when

(a) a 1D GP model with Generalized Wendland correlation and $k = 1, \theta = 0.05$ is fit to data generated from a 1D GP model with Matern correlation with $\nu = 3/2$ and $\theta = 3$, and

(b) a 1D GP model with Generalized Wendland correlation and $k = 3, \theta = 0.10$ is fit to data generated from a 1D GP model with Matern correlation with $\nu = 3/2$ and $\theta = 3$.

Question 5

Congratulations! You are part of the initial human settlement on planet Mars, the Milky Way Galaxy. While life on Mars has been going well, Martian overlord Leon Dusk has decided it is time to expand from the first Martian city, New Frontier, and build new settlements across the planet. One key requirement to kick start this process is to enable the rapid transport of materials and people from New Frontier to various locations around the planet.

You have been tasked with designing a transport rocket for this purpose. Your goal is to design a rocket that has the furthest range. The rocket design program, RocketModeler III (https://www.grc.nasa.gov/www/k-12/rocket/rktsim.html) is the tool you will use in designing your rocket. **You are only allowed to alter the following variables within the given ranges:**

Name	Description	Lower	Upper
X_1	Design→Nose→Length (cm)	0.127	25.147
X_2	Design→Body→Length (cm)	0.0	181.051
X_3	Design→Body→Diameter (cm)	0.249	12.575
X_4	Design→Fins→Length (cm)	1.27	12.585
X_5	Design→Fins→Width (cm)	1.27	6.01
X_6	Pad→Angle (deg)	0.0	60.0

Besides the above variables you can control, the variable Fuel→Endcc should be set to the minimum and Fuel→Begincc should be set to the maximum for all flights. Also set the variable Pad→Location to "Mars" and the Units variable at the top right of the program to "Metric".

You are allowed a *maximum of n = 40* simulator runs. Of these, *a maximum of m = 30* should be used as an initial set of runs selected by a space-filling design. Subsequent runs can be chosen as you see fit with the goal of maximizing the longest range of your rocket design.

(a) Generate your space-filling design for the initial m runs of the simulator and plot the design using the `pairs()` function.

(b) Fit a GP emulator to the observed rocket range outputs. Using your emulator, construct empirical marginal 1-way effect plots for $X_1, \ldots, X_6$.

(c) Perform a sensitivity analysis using your emulator. What variables seem most important in terms of the 1-way sensitivity indices and total sensitivity indices?

(d) Sequentially add the $n - m$ remaining runs using the expected improvement method. For each run added, note if the added run was selected to improve the estimate of the maximum or to reduce the uncertainty of the emulator. Make sure to update your GP emulator after each sequential observation. Plot the final

design of the entire set of n input settings while differentiating which settings were part of the initial design in (a) versus the sequentially added points.

(e) Based on your final GP emulator, what setting $X^* = (X_1^*, \ldots, X_6^*)$ do you recommend as giving the rocket with the longest range? What is the predicted value and 95% confidence interval for the range at input setting X^* using your final GP emulator? What range does the setting X^* achieve when run in the simulator?

(f) Re-run the sensitivity analysis from (c) using your final emulator. What variables seem the most important to designing a long-range rocket?

Note: I recommend, and would prefer, if you use your own GP code for emulation and only resort to the package code for performing the expected improvement updates.
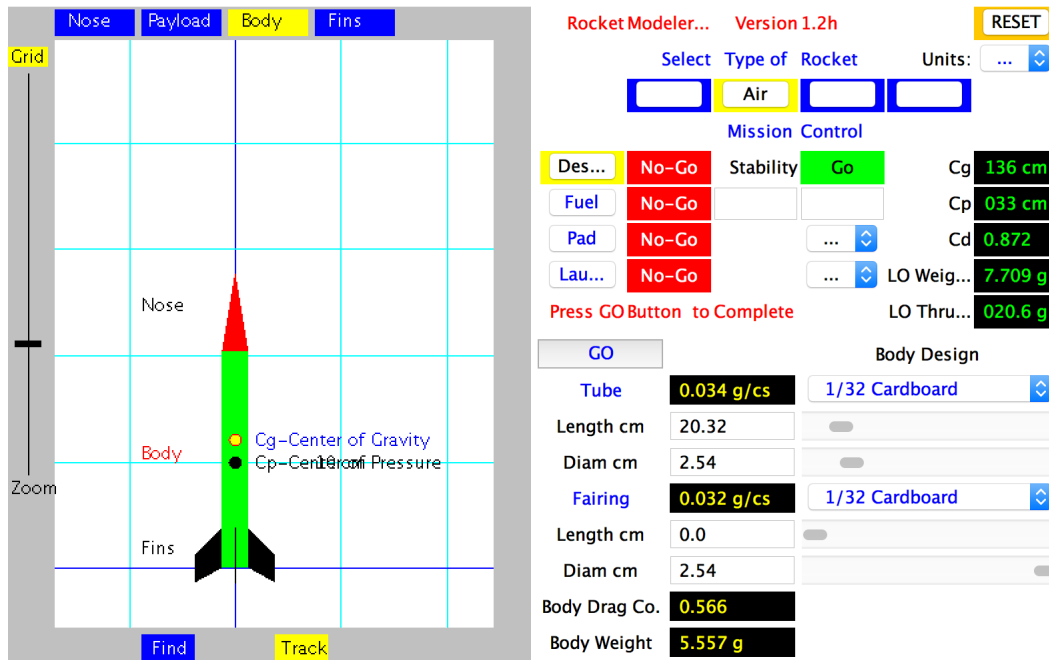

Figure 1: The main screen of RocketModeler III. Make certain the units is set to Metric. The Design, Fuel, Pad and Launch screens are accessed by pressing the buttons located in the center of screen. On the Design screen (shown), the variable settings for Nose, Body and Fins are accessed by pressing the blue buttons at the top left. If you accidentally make a mistake setting variables, hit the Reset button.
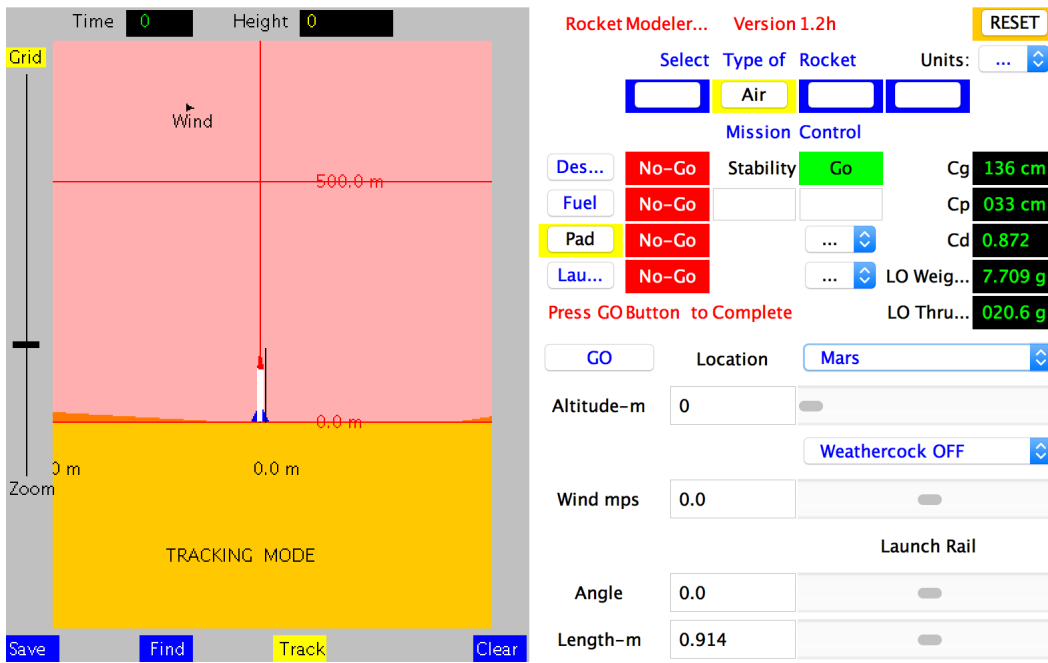
Figure 2: The pad screen of RocketModeler III. Changing the location to "Mars" causes the background behind the rocket to change to a light red landscape as shown.
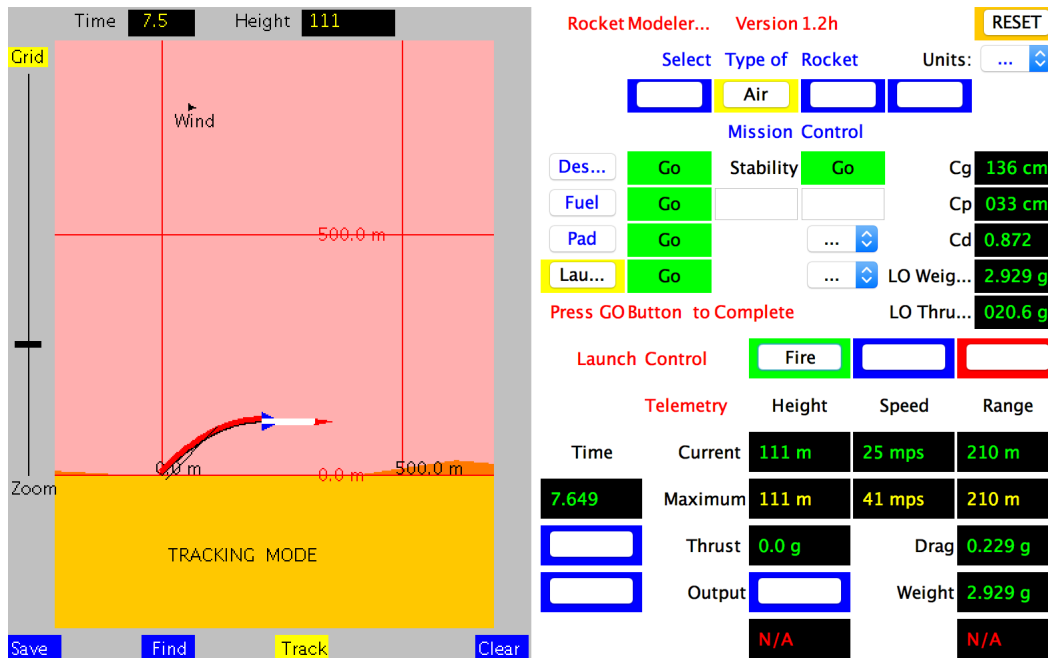

Figure 3: The launch screen of RocketModeler III. Pressing the "Fire" button will launch your rocket. The response variable, Y, is range (downstream distance in meters) travelled by the rocket. This is shown in the black box corresponding to "Current Range". After your rocket reaches land, record this number as your observed response variable. If your design is unstable, the simulator will not allow you to fire your rocket.